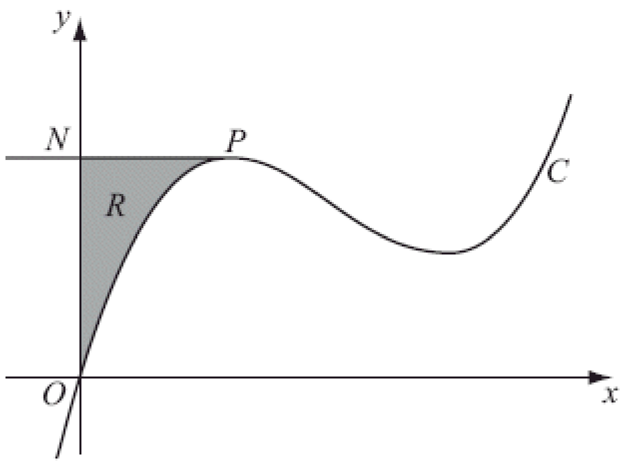
1.



The diagram above shows a sketch of part of the curve C with equation

$$y = x^3 - 10x^2 + kx,$$

where k is a constant.

The point P on C is the maximum turning point.

Given that the x-coordinate of P is 2,

(a) show that $k = 28$.

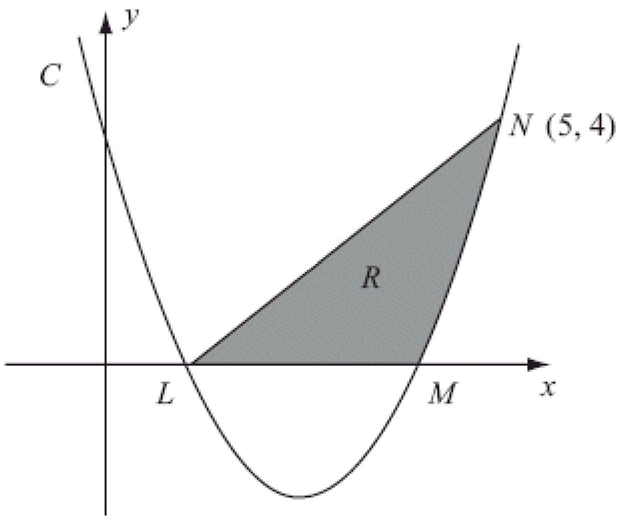
(3)

The line through P parallel to the x-axis cuts the y-axis at the point N. The region R is bounded by C, the y-axis and PN, as shown shaded in the diagram above.

(b) Use calculus to find the exact area of R.

(6)
(Total 9 marks)

2.



The curve C has equation $y = x^2 - 5x + 4$. It cuts the x-axis at the points L and M as shown in the diagram above.

(a) Find the coordinates of the point L and the point M.

(2)

(b) Show that the point N (5, 4) lies on C.

(1)

(c) Find $\int (x^2 - 5x + 4)\,dx$.

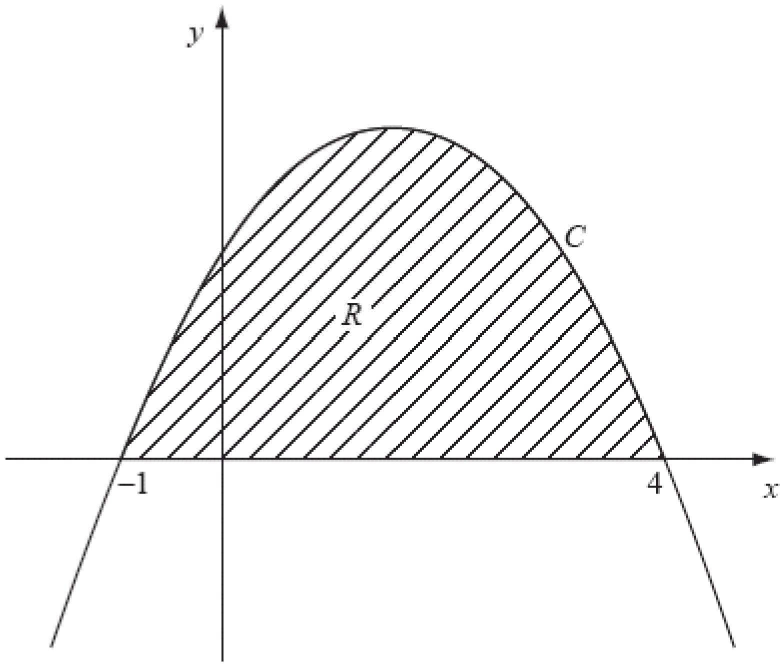
(2)

The finite region R is bounded by LN, LM and the curve C as shown in Figure 2.

(d) Use your answer to part (c) to find the exact value of the area of R.

(5)
(Total 10 marks)

3.



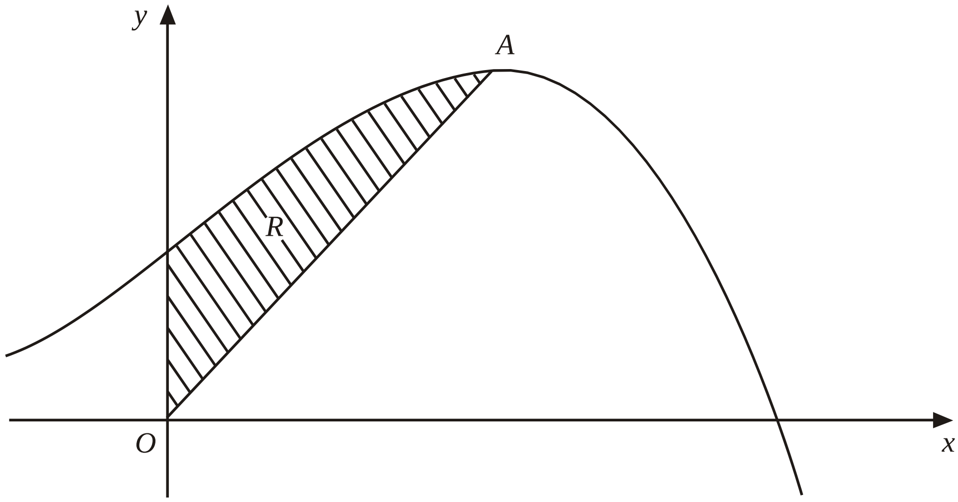
The diagram above shows part of the curve C with equation $y = (1 + x)(4 - x)$.

The curve intersects the x-axis at $x = -1$ and $x = 4$. The region R, shown shaded in the diagram, is bounded by C and the x-axis.

Use calculus to find the exact area of R.

(Total 5 marks)

4.



The diagram above shows a sketch of part of the curve with equation $y = 10 + 8x + x^2 - x^3$.

The curve has a maximum turning point A.

(a) Using calculus, show that the x-coordinate of A is 2.

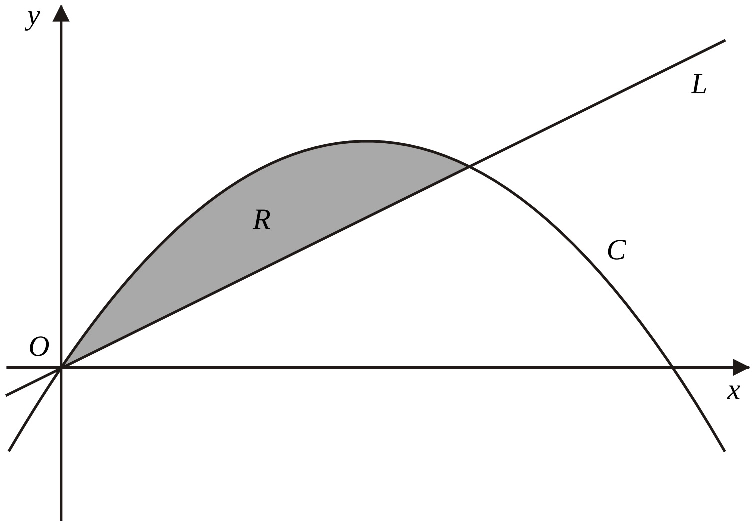
(3)

The region R, shown shaded in the diagram, is bounded by the curve, the y-axis and the line from O to A, where O is the origin.

(b) Using calculus, find the exact area of R.

(8)
(Total 11 marks)

5.



In the diagram above the curve C has equation $y = 6x - x^2$ and the line L has equation $y = 2x$.

(a) Show that the curve C intersects the x-axis at $x = 0$ and $x = 6$.

(1)

(b) Show that the line L intersects the curve C at the points (0, 0) and (4, 8).

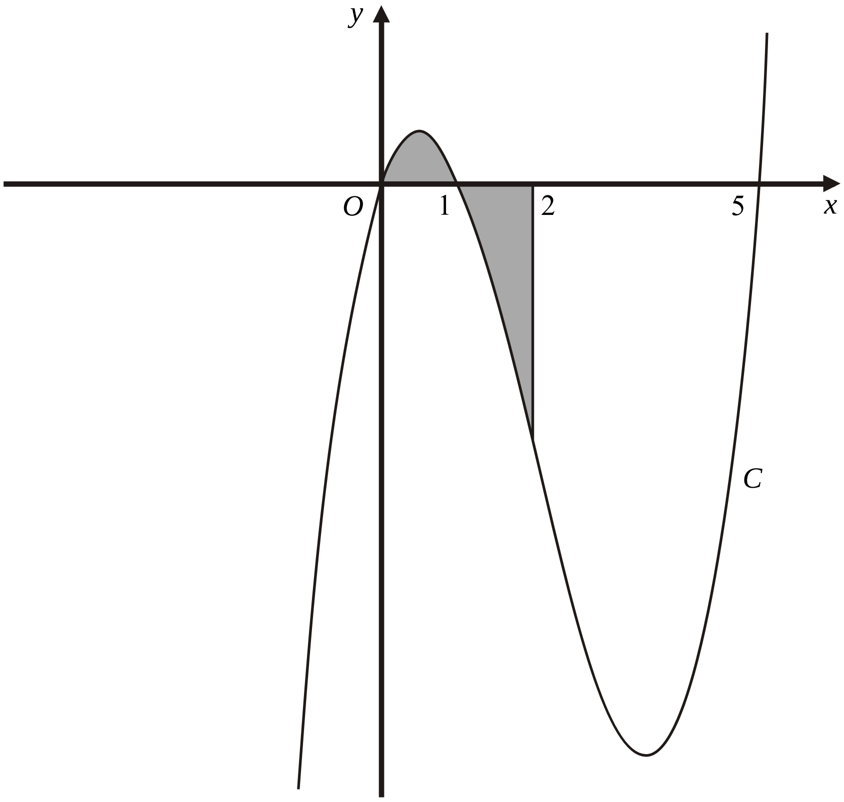
(3)

The region R, bounded by the curve C and the line L, is shown shaded in the diagram above.

(c) Use calculus to find the area of R.

(6)
(Total 10 marks)

6.



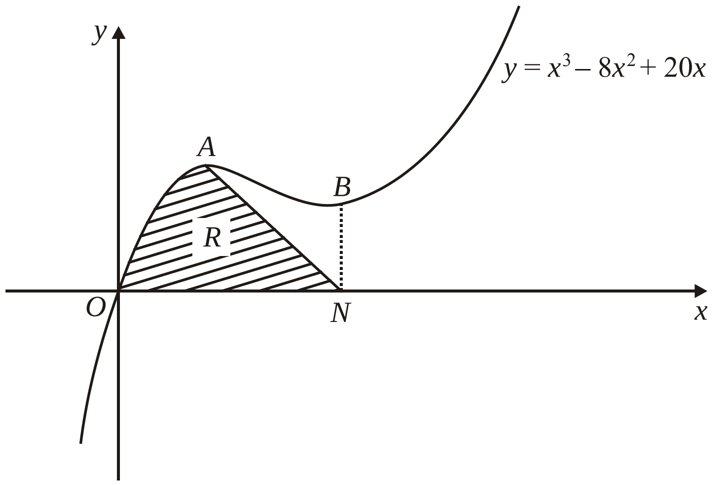
The diagram above shows a sketch of part of the curve C with equation

$$y = x(x - 1)(x - 5).$$

Use calculus to find the total area of the finite region, shown shaded in the diagram, that is between $x = 0$ and $x = 2$ and is bounded by C, the x-axis and the line $x = 2$.

(Total 9 marks)

7.



The figure above shows a sketch of part of the curve with equation $y = x^3 - 8x^2 + 20x$.
The curve has stationary points A and B.

(a) Use calculus to find the x-coordinates of A and B.

(4)

(b) Find the value of $\frac{d^2y}{dx^2}$ at A, and hence verify that A is a maximum.

(2)

The line through B parallel to the y-axis meets the x-axis at the point N.
The region R, shown shaded in the figure above, is bounded by the curve, the x-axis and the line from A to N.

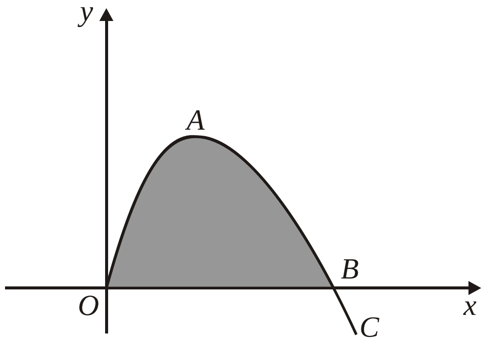
(c) Find $\int (x^3 - 8x^2 + 20x)\,dx$

(3)

(d) Hence calculate the exact area of R.

(5)
(Total 14 marks)

8.



The figure above shows part of the curve C with equation

$$y = 3x^{\frac{1}{2}} - x^{\frac{3}{2}}, \; x \geq 0.$$

The point A on C is a stationary point and C cuts the x-axis at the point B.

(a) Show that the x-coordinate of B is 3.

(1)

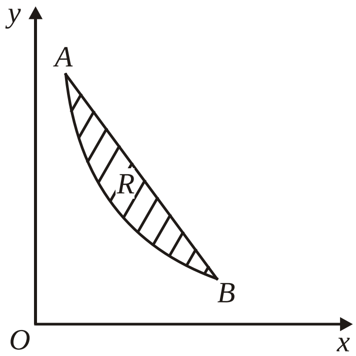
(b) Find the coordinates of A.

(5)

(c) Find the exact area of the finite region enclosed by C and the x-axis, shown shaded in the figure above.

(5)
(Total 11 marks)

9.



The figure above shows the shaded region R which is bounded by the line $y = -2x + 4$ and the curve $y = \frac{3}{2x}$.

The points A and B are the points of intersection of the line and the curve.

Find

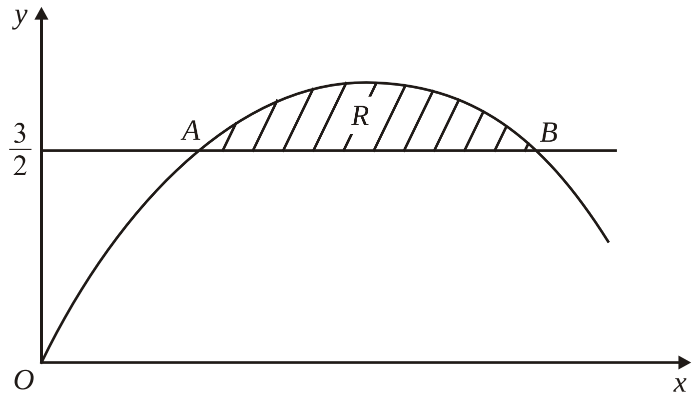
(a) the x-coordinates of the points A and B,

(4)

(b) the exact area of R.

(6)
(Total 10 marks)

10.



The figure above shows the shaded region R which is bounded by the curve $y = -2x^2 + 4x$ and the line $y = \frac{3}{2}$. The points A and B are the points of intersection of the line and the curve.

Find

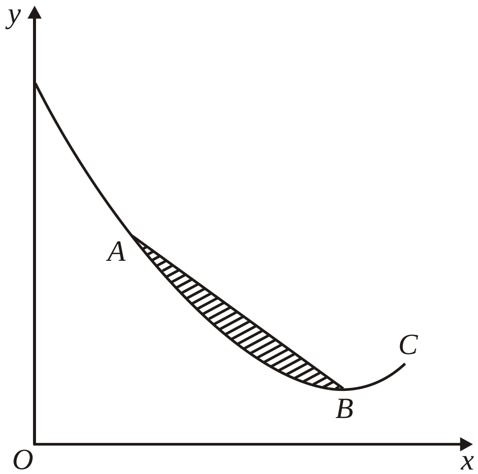
(a) the x-coordinates of the points A and B,

(4)

(b) the exact area of R.

(6)
(Total 10 marks)

11.



This diagram shows part of the curve C with equation

$$y = 2x^{\frac{3}{2}} - 6x + 10, \quad x \geq 0.$$

The curve C passes through the point $A(1, 6)$ and has a minimum turning point at B.

(a) Show that the x-coordinate of B is 4.

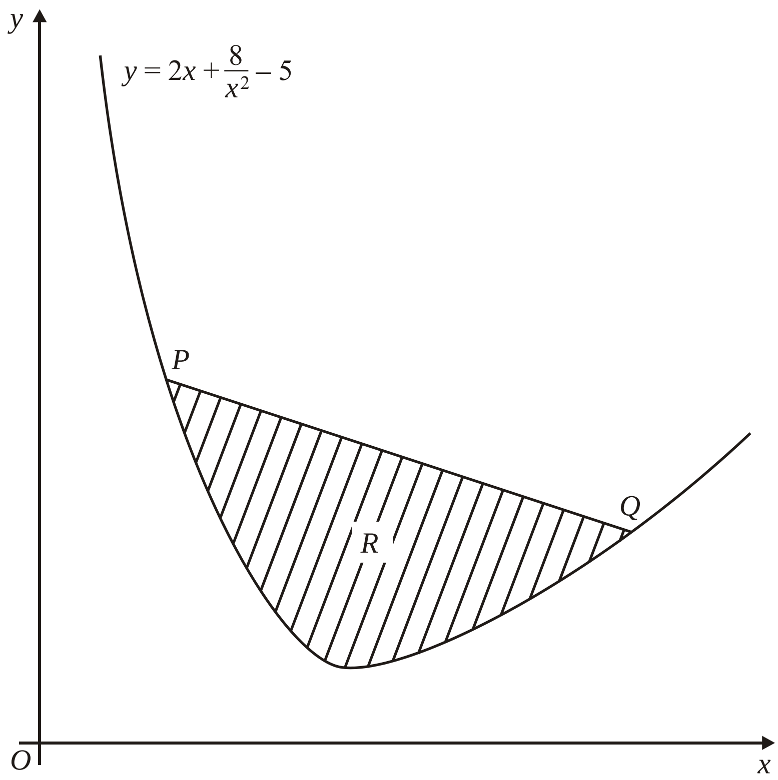
(4)

The finite region R, shown shaded in the diagram, is bounded by C and the straight line AB.

(b) Find the exact area of R.

(8)
(Total 12 marks)

12.



This figure shows part of a curve C with equation $y = 2x + \dfrac{8}{x^2} - 5$, $x > 0$.

The points P and Q lie on C and have x-coordinates 1 and 4 respectively. The region R, shaded in the diagram, is bounded by C and the straight line joining P and Q.

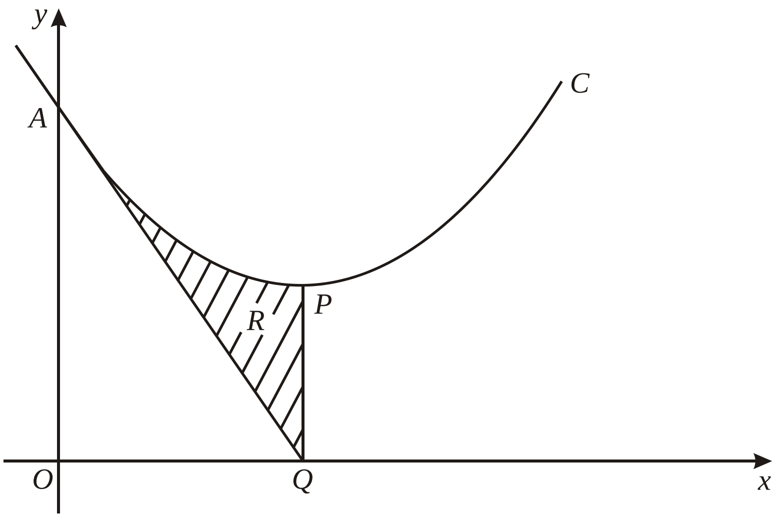
(a) Find the exact area of R.

(8)

(b) Use calculus to show that y is increasing for $x > 2$.

(4)
(Total 12 marks)

13.



The diagram above shows part of the curve C with equation $y = x^2 - 6x + 18$. The curve meets the y-axis at the point A and has a minimum at the point P.

(a) Express $x^2 - 6x + 18$ in the form $(x - a)^2 + b$, where a and b are integers.

(3)

(b) Find the coordinates of P.

(2)

(c) Find an equation of the tangent to C at A.

(4)

The tangent to C at A meets the x-axis at the point Q.

(d) Verify that PQ is parallel to the y-axis.

(1)

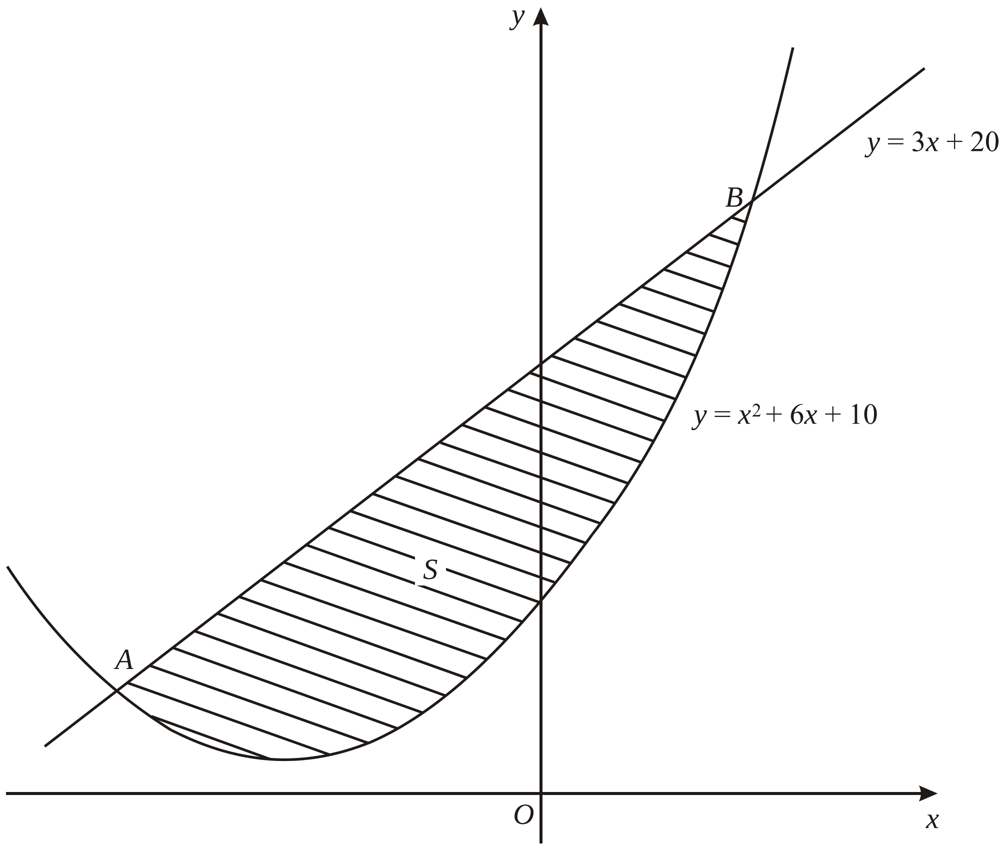
The shaded region R in the diagram is enclosed by C, the tangent at A and the line PQ.

(e) Use calculus to find the area of R.

(5)

(Total 15 marks)

14.



The line with equation $y = 3x + 20$ cuts the curve with equation $y = x^2 + 6x + 10$ at the points A and B, as shown in the diagram.

(a) Use algebra to find the coordinates of A and the coordinates of B.

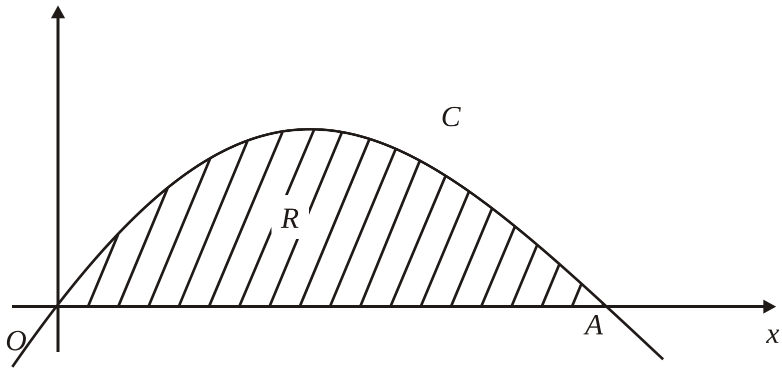
(5)

The shaded region S is bounded by the line and the curve, as shown in the diagram above.

(b) Use calculus to find the exact area of S.

(7)
(Total 12 marks)

15.



The curve C, with equation $y = x(4 - x)$, intersects the x-axis at the origin O and at the point A, as shown in the diagram above. At the point P on C the gradient of the tangent is -2.

(a) Find the coordinates of P.

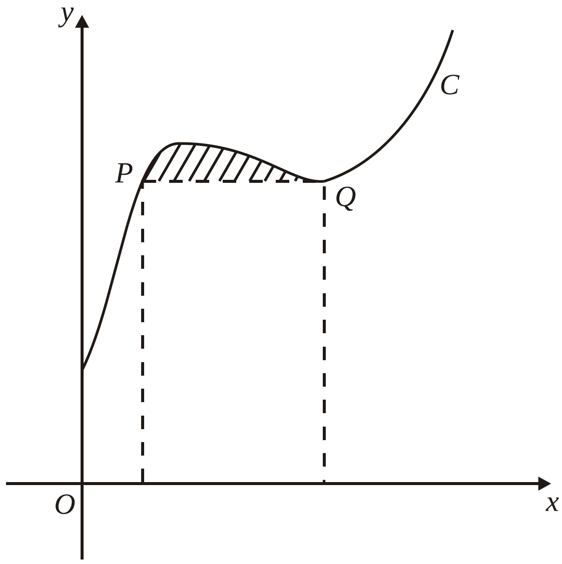
(4)

The region R, enclosed between C and OA, is shown shaded.

(b) Find the exact area of R.

(5)
(Total 9 marks)

16.



The diagram above shows a sketch of part of the curve C with equation

$$y = x^3 - 7x^2 + 15x + 3, \qquad x \geq 0.$$

The point P, on C, has x-coordinate 1 and the point Q is the minimum turning point of C.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

(2)

(b) Find the coordinates of Q.

(4)

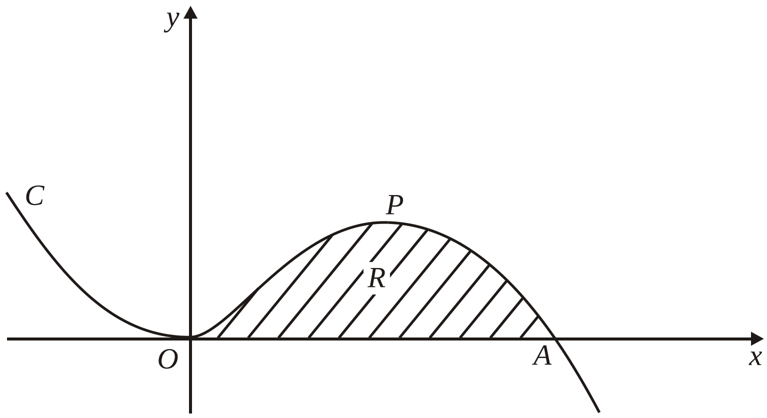
(c) Show that PQ is parallel to the x-axis.

(2)

(d) Calculate the area, shown shaded in the diagram above, bounded by C and the line PQ.

(6)
(Total 14 marks)

17.



The diagram above shows part of the curve C with equation

$$y = \tfrac{3}{2}x^2 - \tfrac{1}{4}x^3.$$

The curve C touches the x-axis at the origin and passes through the point $A(p, 0)$.

(a) Show that $p = 6$.

(1)

(b) Find an equation of the tangent to C at A.

(4)

The curve C has a maximum at the point P.

(c) Find the x-coordinate of P.

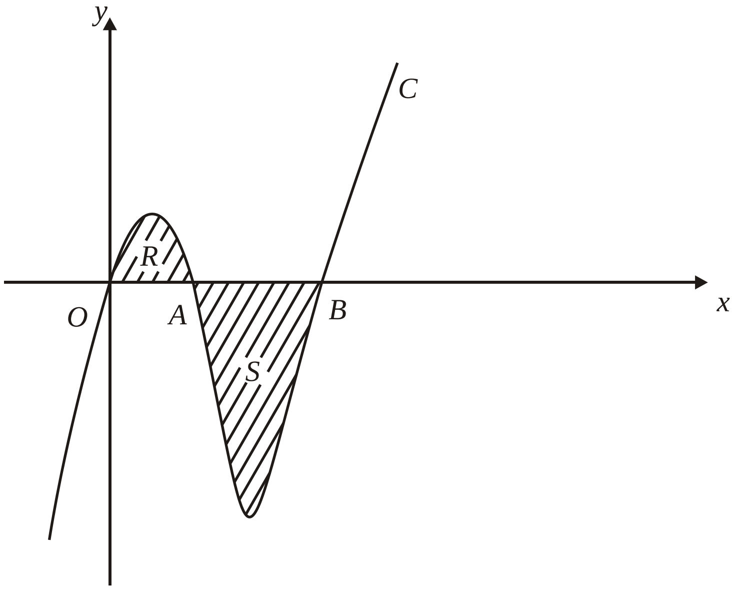
(2)

The shaded region R, in the diagram above, is bounded by C and the x-axis.

(d) Find the area of R.

(4)
(Total 11 marks)

18.



The diagram above shows part of the curve C with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = x^3 - 6x^2 + 5x.$$

The curve crosses the x-axis at the origin O and at the points A and B.

(a) Factorise $\mathrm{f}(x)$ completely.

(3)

(b) Write down the x-coordinates of the points A and B.

(1)

(c) Find the gradient of C at A.

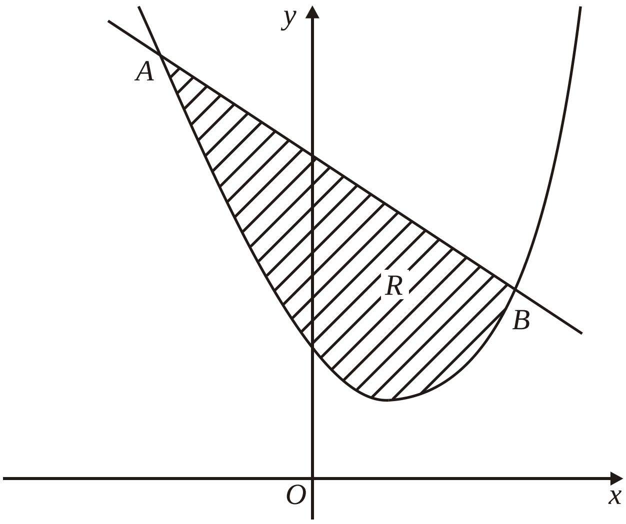
(3)

The region R is bounded by C and the line OA, and the region S is bounded by C and the line AB.

(d) Use integration to find the area of the combined regions R and S, shown shaded in the diagram above.

(7)
(Total 14 marks)

19.



The diagram above shows the line with equation $y = 9 - x$ and the curve with equation $y = x^2 - 2x + 3$. The line and the curve intersect at the points A and B, and O is the origin.

(a) Calculate the coordinates of A and the coordinates of B.

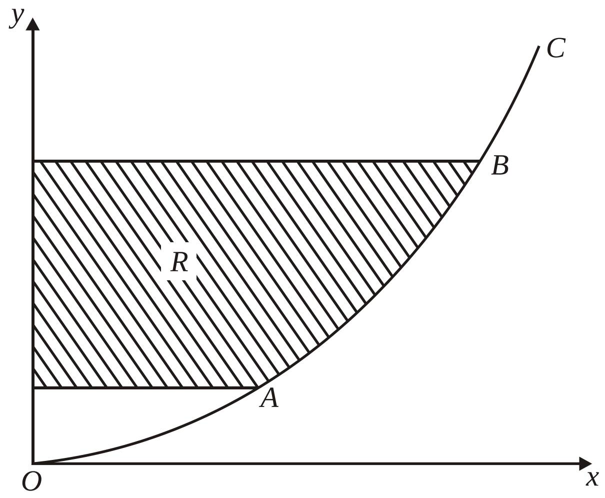
(5)

The shaded region R is bounded by the line and the curve.

(b) Calculate the area of R.

(7)
(Total 12 marks)

20.



The curve C, shown in the diagram above, represents the graph of

$$y = \frac{x^2}{25}, \quad x \geq 0.$$

The points A and B on the curve C have x-coordinates 5 and 10 respectively.

(a) Write down the y-coordinates of A and B.

(1)

(b) Find an equation of the tangent to C at A.

(4)

The finite region R is enclosed by C, the y-axis and the lines through A and B parallel to the x-axis.

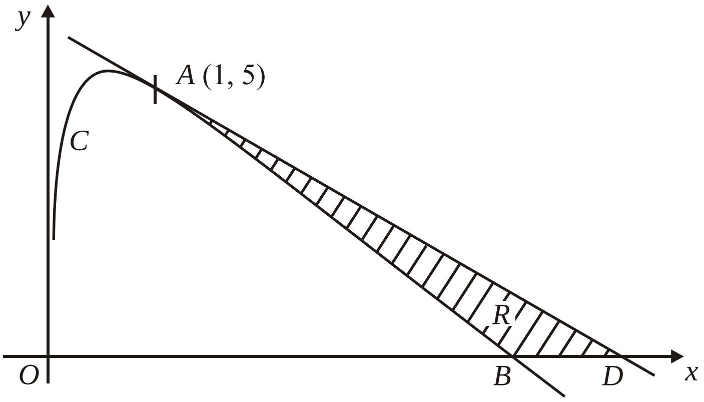
(c) For points (x, y) on C, express x in terms of y.

(2)

(d) Use integration to find the area of R.

(5)
(Total 12 marks)

21.



The diagram above shows part of the curve C with equation

$$y = 9 - 2x - \frac{2}{\sqrt{x}}, \qquad x > 0.$$

The point $A(1, 5)$ lies on C and the curve crosses the x-axis at $B(b, 0)$, where b is a constant and $b > 0$.

(a) Verify that $b = 4$.

(1)

The tangent to C at the point A cuts the x-axis at the point D, as shown in the diagram above.

(b) Show that an equation of the tangent to C at A is $y + x = 6$.

(4)

(c) Find the coordinates of the point D.

(1)

The shaded region R, shown in the diagram above, is bounded by C, the line AD and the x-axis.

(d) Use integration to find the area of R.

(6)
(Total 12 marks)

1. (a) $\frac{dy}{dx} = 3x^2 - 20x + k$ (Differentiation is required) M1 A1

At $x = 2$, $\frac{dy}{dx} = 0$, so $12 - 40 + k = 0$ $k = 28$ (*) A1 cso

N.B. The '= 0' must be seen at some stage to score the final mark.

Alternatively: (using $k = 28$)

$\frac{dy}{dx} = 3x^2 - 20x + 28$ (M1 A1)

'Assuming' $k = 28$ only scores the final cso mark if there is justification that $\frac{dy}{dx} = 0$ at $x = 2$ represents the maximum turning point. 3

Note

M: $x^n \to cx^{n-1}$ (c constant, $c \neq 0$) for one term, seen in part (a).

(b) $\int (x^3 - 10x^2 + 28x)\,dx = \frac{x^4}{4} - \frac{10x^3}{3} + \frac{28x^2}{2}$ Allow $\frac{kx^2}{2}$ for $\frac{28x^2}{2}$ M1 A1

$\left[\frac{x^4}{4} - \frac{10x^3}{3} + 14x^2\right]_0^2 = \ldots$ $\left(= 4 - \frac{80}{3} + 56 = \frac{100}{3}\right)$ M1

(With limits 0 to 2, substitute the limit 2 into a 'changed function')

y-coordinate of P = $8 - 40 + 56 = 24$ Allow if seen in part (a) B1

(The B1 for 24 may be scored by implication from later working)

Area of rectangle = $2 \times (\text{their } y\text{-coordinate of } P)$

Area of R = $(\text{their } 48) - \left(\text{their } \frac{100}{3}\right)$ $= \frac{44}{3}$ $\left(14\frac{2}{3} \text{ or } 14.\dot{6}\right)$ M1 A1

If the subtraction is the 'wrong way round', the final A mark is lost. 6

Note

1st M: $x^n \to cx^{n+1}$ (c constant, $c \neq 0$) for one term.
Integrating the gradient function loses this M mark.

2ndM: Requires use of limits 0 and 2, with 2 substituted into a 'changed function'. (It may, for example, have been differentiated).

Final M: Subtract their values either way round. This mark is dependent on the use of calculus and a correct method attempt for the area of the rectangle.

A1: Must be exact, not 14.67 or similar, but isw after seeing, say, $\frac{44}{3}$.

Alternative: (effectively finding area of rectangle by integration)

$$\int \{24 - (x^3 - 10x^2 + 28x)\} dx = 24x - \left(\frac{x^4}{4} - \frac{10x^3}{3} + \frac{28x^2}{2}\right), \text{ etc.}$$

This can be marked equivalently, with the 1st A being for integrating the same 3 terms correctly. The 3rd M (for subtraction) will be scored at the same stage as the 2nd M. If the subtraction is the 'wrong way round', the final A mark is lost.

[9]

2.	(a)	**Puts** $y = 0$ and attempts to solve quadratic e.g. $(x - 4)(x - 1) = 0$	M1	
		Points are (1,0) and (4, 0)	A1	2

Note

M1 for attempt to find L and M

A1 Accept $x = 1$ and $x = 4$, then isw or accept L =(1,0) , M = (4,0)

Do not accept $L = 1$, $M = 4$ nor (0, 1), (0, 4) (unless subsequent work)

Do not need to distinguish L and M. Answers imply M1A1.

	(b)	$x = 5$ gives $y = 25 - 25 + 4$ and so (5, 4) lies on the curve	B1cso	1

Note

See substitution, working should be shown, need conclusion which could be just $y = 4$ or a tick. Allow $y = 25 - 25 + 4 = 4$ But not $25 - 25 + 4 = 4$. ($y = 4$ may appear at start)

Usually $0 = 0$ or $4 = 4$ is B0

(c) $\int \left(x^2 - 5x + 4\right)\mathrm{d}x = \frac{1}{3}x^3 - \frac{5}{2}x^2 + 4x \quad (+c)$ — M1A1 — 2

<u>Note</u>

M1 for attempt to integrate $x^2 \to kx^3$, $x \to kx^2$ or $4 \to 4x$

A1 for correct integration of all three terms
(do not need constant) isw.

Mark correct work when seen. So e.g. $\frac{1}{3}x^3 - \frac{5}{2}x^2 + 4x$
is A1 then $2x^3 - 15x^2 + 24x$ would be ignored as
subsequent work.

(d) Area of triangle $= \frac{1}{4} \times 4 \times 4 = 8$ or $\int (x-1)\mathrm{d}x = \frac{1}{2}x^2 - x$
with limits 1 and 5 to give 8 — B1

Area under the curve $= \int_4^5$ $\quad \frac{1}{3}\times 5^3 - \frac{5}{2}\times 5^2 + 4\times 5 \quad \left[= -\frac{5}{6}\right]$ — M1

$\frac{1}{3}\times 4^3 - \frac{5}{2}\times 4^2 + 4\times 4 \quad \left[= -\frac{8}{3}\right]$ — M1

$\int_4^5 = -\frac{5}{6} - -\frac{8}{3} = \frac{11}{6}$ or equivalent (allow 1.83 or 1.8 here) — A1 cao

Area of $R = 8 - \frac{11}{6} = 6\frac{1}{6}$ or $\frac{37}{6}$ or 6.16^r
(not 6.17) — A1 cao — 5

<u>Notes</u>

B1 for this triangle only (not triangle *LMN*)

1st M1 for substituting 5 into their changed function

2nd M1 for substituting 4 into their changed function

<u>Alternative method:</u>

$\int_1^5 (x-1) - \left(x^2 - 5x + 4\right)\mathrm{d}x + \int_1^4 x^2 - 5x + 4\mathrm{d}x$
can lead to correct answer

Constructs $\int_1^5 (x-1) - \left(x^2 - 5x + 4\right)\mathrm{d}x$ is B1

M1 for substituting 5 and 1 and subtracting in first integral
M1 for substituting 4 and 1 and subtracting in second integral
A1 for answer to first integral i.e. $\frac{32}{3}$
(allow 10.7) and A1 for final answer as before.

<u>Another alternative</u>

$\int_1^5 (x-1)-(x^2-5x+4)\mathrm{d}x$ + *area of triangle LMP*

Constructs $\int_1^5 (x-1)-(x^2-5x+4)\mathrm{d}x$ is B1

M1 for substituting 5 and 4 and subtracting in first integral

M1 for complete method to find area of triangle (4.5)

A1 for answer to first integral i.e. $\frac{5}{3}$
and A1 for final answer as before.

<u>Could also use</u>

$\int_1^5 (4x-16)-(x^2-5x+4)\mathrm{d}x$ + *area of triangle LMN*

Similar scheme to previous one. Triangle has area 6

A1 for finding Integral has value $\frac{1}{6}$
and A1 for final answer as before.

[10]

3. $y = (1+x)(4-x) = 4 + 3x - x^2$ M: Expand, giving 3 (or 4) terms

$\int (4+3x-x^2)\mathrm{d}x = 4x + \frac{3x^2}{2} - \frac{x^3}{3}$ M: Attempt to integrate

$= [................]^4_{-1} = \left(16+24-\frac{64}{3}\right)-\left(-4+\frac{3}{2}+\frac{1}{3}\right)=\frac{125}{6}$ $\left(=20\frac{5}{6}\right)$ M1 A1 5

<u>Notes</u>

M1 needs expansion, there may be a slip involving a sign or simple arithmetical error e.g. 1 × 4 = 5, but there needs to be a 'constant' an 'x term' and an 'x^2 term'. The x terms do not need to be collected. (Need not be seen if next line correct)

Attempt to integrate means that $x^n \to x^{n+1}$ for at least one of the terms, then **M1** is awarded (even 4 becoming $4x$ is sufficient) – one correct power sufficient.

A1 is for correct answer only, not follow through. But allow $2x^2 - \frac{1}{2}x^2$ or any correct equivalent. Allow + c, and even allow an evaluated extra constant term.

M1: Substitute limit 4 and limit –1 into a changed function (must be –1) and indicate subtraction (either way round).

A1 must be exact, not 20.83 or similar. If recurring indicated can have the mark. Negative area, even if subsequently positive loses the A mark.

Special cases

(i) Uses calculator method: **M1** for expansion (if seen) **M1** for limits if answer correct, so 0, 1 or 2 marks out of 5 is possible (Most likely **M0 M0 A0 M1 A0**)

(ii) Uses trapezium rule : not exact, no calculus – 0/5 unless expansion mark **M1** gained.
(iii) Using original method, but then change all signs after expansion is likely to lead to: **M1 M1 A0, M1 A0 i.e. 3/5**

[5]

4. (a) $\left(\frac{dy}{dx} =\right) 8 + 2x - 3x^2$ (M: $x^n \to x^{n-1}$ for one of the terms, <u>not</u> just $10 \to 0$) M1A1

$3x^2 - 2x - 8 = 0$ $(3x + 4)(x - 2) = 0$ $x = 2$ (Ignore other solution) (*) A1cso 3

The final mark may also be scored by <u>verifying</u> that $\frac{dy}{dx} = 0$ at $x = 2$.

(b) Area of triangle = $\frac{1}{2} \times 2 \times 22$ (M: Correct method to find area of triangle)M1A1

(Area = 22 with no working is acceptable)

$\int 10 + 8x + x^2 - x^2 dx = 10x + \frac{8x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$

(M: $x^n \to x^{n+1}$ for one of the terms) M1A1A1

Only one term correct: M1A0A0
2 or 3 terms correct: M1A1A0
Integrating the <u>gradient function</u> loses this M mark.

$\left[10x + \frac{8x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}\right]_0^2 = \ldots\ldots$ (Substitute limit 2 into a 'changed function') M1

$\left(= 20 + 16 + \frac{8}{3} - 4\right)$ (This M can be awarded even if the other limit is wrong)

Area of R = $34\frac{2}{3} - 22 = \frac{38}{3}\left(= 12\frac{2}{3}\right)$ (Or $12.\dot{6}$) M1A1 8

M: <u>Dependent on use of calculus in (b) and correct overall 'strategy'</u>: subtract either way round.

A: Must be <u>exact</u>, not 12.67 or similar. A negative area at the end, even if subsequently made positive, loses the A mark.

<u>Alternative</u>:
Eqn. of line $y = 11x$. (Marks dependent on subsequent use in integration) M1A1
(M1: Correct method to find equation of line.
A1: Simplified form $y = 11x$)

$\int 10 + kx + x^2 - x^3 dx = 10x + \frac{kx^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$ (k perhaps –3) M1A1A1

$\left[10x + \frac{kx^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}\right]_0^2 = \ldots\ldots.$ (Substitute limit 2 into a 'changed function') M1

Area of R = $\left[10x - \frac{3x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}\right]_0^2 = 20 - 6 + \frac{8}{3} - 4 = \frac{38}{3}\left(= 12\frac{2}{3}\right)$ M1A1 8

Final M1 for $\int$(curve) – $\int$(line) or $\int$(line) – $\int$(curve).

[11]

5.	(a)	Either solving $0 = x(6 - x)$ and showing $x = 6$ (and $x = 0$)	B1	1
	(b)	or showing (6, 0) (and $x = 0$) satisfies $y = 6x - x^2$ [allow for showing $x = 6$]		
		Solving $2x = 6x - x^2$ ($x^2 = 4x$) to $x = \ldots.$	M1	
		$x = 4$ (and $x = 0$)	A1	
		Conclusion: when $x = 4$, $y = 8$ and when $x = 0$, $y = 0$,	A1	3

In scheme first A1: need only give $x = 4$
If *verifying approach* used:
Verifying (4,8) satisfies both the line and the curve M1(attempt at both),
Both shown successfully A1
For final A1, (0,0) needs to be mentioned ; accept "clear from diagram"

	(c)	(Area =) $\int_{(0)}^{(4)} (6x - x^2)\,dx$ Limits not required	M1	
		Correct integration $3x^2 - \frac{x^3}{3}$ (+ c)	A1	
		Correct use of correct limits on their result above (see notes on limits)	M1	
		$["3x^2 - \frac{x^3}{3}"]^4 - ["3x^2 - \frac{x^3}{3}"]_0$ with limits substituted $[= 48 - 21\frac{1}{3} = 26\frac{2}{3}]$		
		Area of triangle = 2 × 8 =16 (Can be awarded even if no M scored, i.e. B1)	A1	
		Shaded area = ± (area under curve – area of triangle) applied correctly	M1	
		$(= 26\frac{2}{3} - 16) = 10\frac{2}{3}$ (awrt 10.7)	A1	6

Alternative Using Area = $\pm\int_{(0)}^{(4)} \{(6x - x^2) ; -2x\}\,dx$ approach

(i) If candidate integrates separately can be marked as main scheme
If combine to work with = $\pm\int_{(0)}^{(4)} (4x - x^2)\,dx$,
first M mark and **third** M mark
= $(\pm)[2x^2 - \frac{x^3}{3} (+c)]$ A1,
Correct use of correct limits on their result **second** M1,
Totally correct, unsimplified ± expression (may be implied by correct ans.) A1
$10\frac{2}{3}$ A1 [Allow this if, having given – $10\frac{2}{3}$, they correct it]

M1 *for correct use of correct limits:* Must substitute correct limits for their strategy into a changed expression and subtract, either way round, e.g $\pm\{[\quad]^4 - [\quad]_0\}$

If a long method is used, e,g, finding three areas, this mark only gained for correct strategy and all limits need to be correct for this strategy.
Final M1: limits for area under curve and triangle must be the same.

S.C. (1) $\int_0^6 (6x - x^2)\mathrm{d}x - \int_0^6 2x\mathrm{d}x = \left[3x^2 - \frac{x^3}{3}\right]_0^6 - \left[x^2\right]_0^6$ = ... award M1A1

MO(limits)AO(triangle)M1(bod)A0

(2) If, having found ± correct answer, thinks this is not complete strategy and does more, do not award final 2 A marks

Use of trapezium rule: M0A0MA0 possible A1 for triangle
M1 (if correct application of trap. rule from $x = 0$ to $x = 4$) A0

[10]

6. $y = x(x^2 - 6x + 5)$

$= x^3 - 6x^2 + 5x$ M1, A1

$\int (x^3 - 6x^2 + 5x)\mathrm{d}x = \frac{x^4}{4} - \frac{6x^3}{3} + \frac{5x^3}{2}$ M1, A1ft

$\left[\frac{x^4}{4} - 2x^3 + \frac{5x^2}{2}\right]_0^1 = \left(\frac{1}{4} - 2 + \frac{5}{2}\right) - 0 = \frac{3}{4}$ M1

$\left[\frac{x^4}{4} - 2x^3 + \frac{5x^2}{2}\right]_1^2 = (4 - 16 + 10) - \frac{3}{4} = -\frac{11}{4}$ M1, A1(both)

$\therefore$ total area = $\frac{3}{4} + \frac{11}{4}$ M1

$= \frac{7}{2}$ o.e. A1cso 9

Attempt to multiply out, must be a cubic. M1

Award A mark for their final version of expansion (but final version does not need to have like terms collected). A1

Attempt to integrate; $x^n \to x^{n+1}$. Generous mark for some use of integration, so e.g. $\int x(x-1)(x-5)\mathrm{d}x = \frac{x^2}{2}\left(\frac{x^2}{2} - x\right)\left(\frac{x^2}{2} - 5x\right)$ would gain method mark. M1

Ft on their final version of expansion provided it is in the form $ax^p + bx^q + ...$
Integrand must have at least two terms and all terms must be integrated correctly.
If they integrate twice (e.g. $\int_0^1$ and $\int_1^2$) and get different answers, take the better of the two. A1ft

Substitutes and subtracts (either way round) for one integral.
Integral must be a 'changed' function. Either 1 and 0, 2 and 1 or 2 and 0.
For $[\ ]_0^1$: – 0 for bottom limit can be implied (provided that it is 0). M1

M1 Substitutes and subtracts (either way round) for two integrals.

Integral must be a 'changed' function. Must have 1 and 0 and 2 and 1 (or 1 and 2).
The two integrals do not need to be the same, but they must have come from attempts to integrate the same function. M1

$\frac{3}{4}$ and $-\frac{11}{4}$ o.e. (if using $\int_1^2 f(x)$) or $\frac{3}{4}$ and $\frac{11}{4}$ or (if using $\int_2^1 f(x)$ or $-\int_1^2 f(x)$ or $\int_1^2 -f(x)$) where $f(x) = \frac{x^4}{4} - 2x^3 + \frac{5x^2}{2}$.
The answer must be consistent with the integral they are using (so $\int_1^2 f(x) = \frac{11}{4}$ loses this A and the final A).
$-\frac{11}{4}$ may not be seen explicitly.
Can be implied by a subsequent line of working. A1

5^{th} M1 | their value for $[\ ]_0^1$ |+| their value for $[\ ]_1^2$ |
Dependent on at least one of the values coming from integration (other may come from e.g. trapezium rules).
This can be awarded even if both values already positive. M1

$\frac{7}{2}$ o.e. N.B. c.s.o. A1 cso

[9]

7. (a) $\frac{dy}{dx} = 3x^2 - 16x + 20$ M1 A1

$3x^2 - 16x + 20 = 0$ $(3x - 10)(x - 2) = 0$ $x = ..., \frac{10}{3}$ and 2 dM1 A1 4

The second M is dependent on the first, and requires an attempt to solve a 3 term quadratic.

(b) $\frac{d^2y}{dx^2} = 6x - 16$ At $x = 2$, $\frac{d^2y}{dx^2} = ...$ M1

-4 (or < 0, or both), therefore maximum A1ft 2

M1: Attempt second differentiation and substitution of one of the x values.

A1ft: Requires correct second derivative and negative value of the second derivative, but ft from their x value.

(c) $\int (x^3 - 8x^2 + 20x)\,dx = \frac{x^4}{4} - \frac{8x^3}{3} + \frac{20x^2}{2} (+C)$ M1 A1 A1 3

All 3 terms correct: M1 A1 A1,
Two terms correct: M1 A1 A0,
One power correct: M1 A0 A0.

(d) $4-\frac{64}{3}+40$ $\left(=\frac{68}{3}\right)$ M1

A: $x = 2$: $y = 8 - 32 + 40 = 16$ (May be scored elsewhere) B1

Area of $\Delta = \frac{1}{2}\left(\frac{10}{3}-2\right)\times 16$ $\left(\frac{1}{2}(x_B - x_A)\times y_A\right)$ $\left(=\frac{32}{3}\right)$ M1

Shaded area $= \frac{68}{3}+\frac{32}{3}=\frac{100}{3}$ $\left(=33\frac{1}{3}\right)$ M1 A1 5

Limits M1: Substituting their lower x value into a 'changed' expression.

Area of triangle M1: Fully correct method.

Alternative for the triangle (finding an equation for the straight line then integrating) requires a fully correct method to score the M mark.

Final M1: Fully correct method (beware valid alternatives!)

[14]

8. (a) $y = 0 \Rightarrow x^{\frac{1}{2}}(3-x) = 0 \Rightarrow x = 3$ B1 1

or $3\sqrt{3} - 3^{\frac{3}{2}} = 3\sqrt{3} - 3\sqrt{3} = 0$

(b) $\frac{dy}{dx} = \frac{3}{2}x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}}$ $x^n \mapsto x^{n-1}$ M1 A1

$\frac{dy}{dx} = 0 \Rightarrow x^{\frac{1}{2}} = x^{-\frac{1}{2}}$ Use of $\frac{dy}{dx} = 0$ M1

$\Rightarrow x = 1$ A1

A: (1, 2) A1 5

(c) $\int\left(3x^{\frac{1}{2}} - x^{\frac{3}{2}}\right)dx = 2x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}}$ M1 $x^n \mapsto x^{n+1}$ M1 A1+A1

Accept unsimplified expressions for As

Area $= [...]_0^3 = 2\times 3\sqrt{3} - \frac{2}{5}\times 9\sqrt{3}$ Use of correct limits M1

Area is $\frac{12}{5}\sqrt{3}$ (units^2) A1 5

For final A1, terms must be collected together but accept exact equivalents, e.g. $\frac{4}{5}\sqrt{27}$

[11]

9. (a) $-2x + 4 = \frac{3}{2x}$ M1

$4x^2 - 8x + 3 = 0$ A1

$(2x - 3)(2x - 1) = 0$ M1

$x = 0.5, 1.5$ A1 4

(b) $\int_{0.5}^{1.5} -2x + 4 dx = \left[-x^2 + 4x\right]_{0.5}^{1.5}$ or $\frac{1}{2} \times (3 + 1) \times 1$ M1

$= 2$ A1

$\int_{0.5}^{1.5} \frac{3}{2x} dx = \left[\frac{3}{2}\ln x\right]_{0.5}^{1.5}$ M1 A1

$= \frac{3}{2}\ln 3$ A1ft

$\therefore$ Area $= 2 - \frac{3}{2} \ln 3$ A1 6

Alternative solution:

Area $= \int_{0.5}^{1.5} -2x + 4 - \frac{3}{2x} dx$ M1

$= \left[-x^2 + 4x - \frac{3}{2}\ln x\right]_{0.5}^{1.5}$ M1 A1 A1

$= \frac{-9}{4} + \frac{1}{4} + 6 - 2 - \frac{3}{2}\ln 3$ o.e. A1ft

$= 2 - \frac{3}{2} \ln 3$ A1

[10]

10. (a) $\frac{3}{2} = -2x^2 + 4x$ M1

$4x^2 - 8x + 3 (= 0)$ A1

$(2x - 1)(2x - 3) = 0$ M1

$\underline{x = \frac{1}{2}, \quad \frac{3}{2}}$ A1 4

(b) Area of $R = \int_{\frac{1}{2}}^{\frac{3}{2}} (-2x^2 + 4x)\, dx \quad -\frac{3}{2}$ (for $-\frac{3}{2}$) B1

$\int \left(-2x^2 + 4x\right) \ dx = \left[-\frac{2}{3}x^3 + 2x^2\right]$ (Allow ±[ ], accept $\frac{4}{2}x^2$ M1 [A1]

$$\int_{\frac{1}{2}}^{\frac{3}{2}} (-2x^2 + 4x)\,dx = \left(-\frac{2}{3}\times\frac{3^3}{2^3} + 2\times\frac{3^2}{2^2}\right) -, \quad \left(-\frac{2}{3}\times\frac{1}{2^3} + 2\times\frac{1}{2^2}\right)$$ M1 M1

$$\left(=\frac{11}{6}\right)$$

Area of $R = \frac{11}{6} - \frac{3}{2} = \underline{\frac{1}{3}}$ (Accept exact equivalent but not 0.33 ...) A1cao 6

[10]

(a) 1st M1 for forming a correct equation
1st A1 for a correct 3TQ (condone missing = 0 but must have all terms on one side)
2nd M1 for attempting to solve appropriate 3TQ

(b) B1 for subtraction of $\frac{3}{2}$. Either "curve – line" or "integral – rectangle"

1st M1 for some correct attempt at integration ($x^n \to x^{n+1}$)

1st A1 for $-\frac{2}{3}x^3 + 2x^2$ only i.e. can ignore $-\frac{3}{2}x$

2nd M1 for some correct use of their $\frac{3}{2}$ as a limit in integral

3rd M1 for some correct use of their $\frac{1}{2}$ as a limit in integral and subtraction either way round

Special Case Line – curve gets B0 but can have the other A marks provided final answer is $+\frac{1}{3}$.

11. (a) $\frac{dy}{dx} = 3x^{1/2} - 6$ M1 A1

$3x^{1/2} - 6 = 0,\ x^{1/2} = 2 \qquad x = 4$ (*) M1 A1 4

First M1 for decrease of 1 in power of x of at least one term (disappearance of "10" sufficient)

Second M1 for putting $\frac{dy}{dx}$ = 0 and finding x = ….

(b) $\int\left(2x^{3/2} - 6x + 10\right) dx = \left[\frac{4x^{5/2}}{5} - 3x^2 + 10x\right]$ M1 A1 A1

$\left[\frac{4x^{5/2}}{5} - 3x^2 + 10x\right]_1^4 = \left(\frac{4\times 4^{5/2}}{5} - (3\times 16) + 40\right) - \left(\frac{4}{5} - 3 + 10\right)$ M1 A1ft

(= 17.6 – 7.8 = 9.8)

Finding area of trapezium = $\frac{1}{2}(6 + 2) \times 3$ (=12) M1 A1

[A = (1, 6), B = (4, 2)]

Or by integration: $\left[\frac{22x - 2x^2}{3}\right]_1^4$

Area of R =12 – 9.8 = 2.2 A1 8

First M1: Power of at least one term increased by 1

First A1: For $\frac{4x^{5/2}}{5}$

Second A1: For $-3x^2 + 10x$

Second M1 for limits requires $|[\ ]^4 - [\ ]_1|$ *(allow candidate's "4")*

and some processing of "integral", $[y]_1^4$ *is M0*

A1ft requires ***1*** *and* ***4*** *substituted in candidate's* ***3-termed integrand*** *(unsimplified)*

Area of trapezium: M1 attempt at ½ $(y_A + y_B)(x_B - 1)$

or $\int \frac{22-4x}{3}dx$ *(A1 correct unsimplified)*

EXTRA

Attempting integral |(equation of line – equation of curve)! Third M1

$= \int |(-\frac{8}{3} + \frac{14}{3}x - 2x^{\frac{3}{2}})|\,dx$ Fourth A1

Performing integration: First M1

$\left[(-\frac{8}{3}x + \frac{7}{3}x^2) - (\frac{4}{5}x^{\frac{5}{2}})\right]$ $\left[(\frac{4}{5}x^{\frac{5}{2}})\right]$ First A1

$\left[|(-\frac{8}{3}x + \frac{7}{3}x^2)|\right]$ allow as follow through in this case. Second A1

Limits M1A1$\sqrt{\ }$ Second M1

Third A1

Answer A1 Fifth A1

[12]

12. (a) $\int(2x + 8x^{-2} - 5)dx = x^2 + \frac{8x^{-1}}{-1} - 5x$ M1 A1 A1

$\left[x^2 + \frac{8x^{-1}}{-1} - 5x\right]_1^4 = (16 - 2 - 20) - (1 - 8 - 5) \quad (= 6)$ M1

$x = 1$: $y = 5$ and $x = 4$: $y = 3.5$ B1

Area of trapezium = $\frac{1}{2}(5 + 3.5)(4 - 1) \quad (= 12.75)$ M1

Shaded area = 12.75 – 6 = 6.75 M1 A1 8

(M: Subtract either was round)

Integration: One term wrong M1 A1 A0; two terms wrong M1 A0 A0.

Limits: M1 for substituting limits 4 and 1 into a changed function, and subtracting the right way round.

Alternative:

$x = 1$: $y = 5$ and $x = 4$: $y = 3.5$ B1

Equation of line: $y - 5 = -\frac{1}{2}(x - 1)$ $\quad y = \frac{11}{2} - \frac{1}{2}x$, subsequently used in integration with limits. 3rd M1

$\left(\frac{11}{2}-\frac{1}{2}x\right)-\left(2x+\frac{8}{x^2}-5\right)$ 4th M1

(M: Subtract either way round)

$\int\left(\frac{21}{2}-\frac{5x}{2}-8x^{-2}\right)\mathrm{d}x=\frac{21x}{2}-\frac{5x^2}{4}-\frac{8x^{-1}}{-1}$ 1sr M1 A1ft A1ft

(Penalise integration mistakes, not algebra for the ft marks)

$\left[\frac{21x}{2}-\frac{5x^2}{4}-\frac{8x^{-1}}{-1}\right]_1^4=(42-20+2)-\left(\frac{21}{2}-\frac{5}{4}+8\right)$ 2nd M1

(M: Right way round)

Shaded area = 6.75 A1

(The follow through marks are for the subtracted version, and again deduct an accuracy mark for a wrong term: One wrong M1 A1 A0; two wrong M1 A0 A0.)

(b) $\frac{\mathrm{d}y}{\mathrm{d}x}=2-16x^{-3}$ M1 A1

(Increasing where) $\frac{\mathrm{d}y}{\mathrm{d}x}>0$; For $x>2$, $\frac{16}{x^3}<2$, $\therefore$ $\frac{\mathrm{d}y}{\mathrm{d}x}>0$ dM1; A1 4

(allow ≥)

<u>Alternative for the last 2 marks in (b)</u>:

M1: Show that $x = 2$ is a minimum, using, e.g., 2nd derivative.
A1: Conclusion showing understanding of " increasing", with accurate working.

[12]

13. (a) $(x-3)^2$, +9 isw . $a = 3$ and $b = 9$ may just be written down with no method shown. B1, M1 A1 3

(b) P is (3, 9) B1

(c) A = (0, 18) B1

$\frac{\mathrm{d}y}{\mathrm{d}x}=2x-6$, at A $m=-6$ M1 A1

Equation of tangent is $y-18=-6x$ (in any form) A1ft 4

(d) Showing that line meets x axis directly below P, i.e. at $x = 3$. A1cso 1

(e) $A=\int x^2-6x+18x=[\frac{1}{3}x^3-3x^2+18x]$ M1 A1

Substituting $x = 3$ to find area A under curve A [=36] M1

Area of $R = A$ – area of triangle = $A-\frac{1}{2}\times 18\ 3$, = 9 M1 A1 5

Alternative: $\int x^2-6x+18-(18-6x)\mathrm{d}x$ M1

$=\frac{1}{3}x^3$ M1 A1 ft

Use $x = 3$ to give answer 9 M1 A1

[13]

14.	(a)	$x^2 + 6x + 10 = 3x + 20$	M1	
		$\Rightarrow x^2 + 3x - 10 = 0$		
		$(x + 5)(x - 2) = 0$ so $x =, -5$ or 2	M1, A1	
		sub a value for x to obtain a value for y in $y = 3x + 20$, $y = 5$ or 26	M1, A1	5
	(b)	line – curve =, $10 - 3x - x^2$	M1, A1	
		$\int (10 - 3x - x^2) dx = 10x - \frac{3}{2}x^2 - \frac{x^3}{3}$	M1 A2/1/0f.t.	
		$\left[10x - \frac{3}{2}x^2 - \frac{x^3}{3}\right]_{-5}^{2} = (20 - \frac{3}{2}\times 4 - \frac{8}{3}) - (-50 - \frac{3}{2}\times 25 + \frac{125}{3})$	M1	
		$= 11\frac{1}{3} - -45\frac{5}{6} = \underline{\underline{57\frac{1}{6}}}$	A1	7

[12]

ALT (b)	$\int (x^2 + 6x + 10) dx = \frac{x^3}{3} + 3x^2 + 10x$ (–1 each incorrect term)	M1 A2/1/0	
	use of limits = $(\frac{8}{3} + 12 + 20) - (-\frac{125}{3} + 75 - 50) = (51\frac{1}{3})$	M1	
	Area of Trapezium = $\frac{1}{2}(5 + 26)(2 - -5) = (108\frac{1}{2})$		
	or $46 - -62.5$ (from integration)	B1f.t.	
	Shaded area = Trapezium – $\int = 108\frac{1}{2} - 51\frac{1}{3} = 57\frac{1}{6}$ or $\frac{343}{6}$		
	or $57.1\dot{6}$	M1 A1	7

15.	(a)	$y = 4x - x^2$ $\quad \frac{dy}{dx} = 4 - 2x$	M1 A1	
		"$4 - 2x$" $= -2$, $\quad x = \ldots$	M1	
		$x = 3$, $y = 3$	A1	4
	(b)	x-coordinate of A is 4	B1	
		$\int (4x - x^2) dx = \left[\frac{4x^2}{2} - \frac{x^3}{3}\right]$	M1 A1ft	
		$\left[\frac{4x^2}{2} - \frac{x^3}{3}\right]_0^4 = 32 - \frac{64}{3} = \frac{32}{3}$ $\quad \left(= 10\frac{2}{3}\right)$ (or exact equivalent)	M1 A1	5

[9]

16. (a) $\frac{dy}{dx} = 3x^2 - 14x + 15$ M1 A1 2

(b) $3x^2 - 14x + 15 = 0$ M1

$(3x - 5)(x - 3) = 0 \quad x = \ldots,$ M1, A1

(A1 requires correct quadratic factors).

$y = 12$ A1 4

(Following from x = 3)

(c) P: $x = 1 \quad y = 12$ B1

Same y-coord. as Q (or "zero gradient"), so PQ is parallel to the x-axis B1 2

(d) $\int (x^3 - 7x^2 + 15x + 3)dx = \frac{x^4}{4} - \frac{7x^3}{3} + \frac{15x^2}{2} + 3x$ M1 A1 A1

(First A1: 3 terms correct, Second A1: all correct)

$\left[\frac{x^4}{4} - \frac{7x^3}{3} + \frac{15x^2}{2} + 3x\right]_1^3 = \left(\frac{81}{4} - 63 + \frac{135}{2} + 9\right) - \left(\frac{1}{4} - \frac{7}{3} + \frac{15}{2} + 3\right)$ M1

$\left(33\frac{3}{4} - 8\frac{5}{12}\right) - 24 = 25\frac{1}{3} - (2 \times 12) = 1\frac{1}{3}$ M1 A1 6

(or equiv. or 3 s.f or better)

[14]

17. (a) Solve $\frac{3}{2}x^2 - \frac{1}{4}x^3 = 0$ to find $p = 6$, or verify: $\frac{3}{2} \times 6^2 - \frac{1}{4} \times 6^3 = 0$ (*) B1 1

(b) $\frac{dy}{dx} = 3x - \frac{3x^2}{4}$ M1 A1

$m = -9$, $y - 0 = -9(x - 6)$ (Any correct form) M1 A1 4

(c) $3x - \frac{3x^2}{4} = 0$, $x = 4$ M1, A1ft 2

(d) $\int \left(\frac{3x^2}{2} - \frac{x^3}{4}\right)dx = \frac{x^3}{2} - \frac{x^4}{16}$ (Allow unsimplified versions) M1 A1

$[\ldots\ldots\ldots]_0^6 = \frac{6^3}{2} - \frac{6^4}{16} = 27$ M: Need 6 and 0 as limits. M1 A1 4

[11]

18. (a) Correct method for one of the 3 factors. M1

$x(x-1)(x-5)$ M1 A1 3

Allow $(x \pm 0)$ instead of x.
(2^{nd} M1 for attempting full factorisation)

(b) 1 and 5 B1 ft 1

(c) $\frac{dy}{dx} = 3x^2 - 12x + 5$ M1 A1

At $x = 1$. $\frac{dy}{dx} = 3 - 12 + 5 = -4$ A1 3

(d) $\int (x^3 - 6x^2 + 5x)dx = \frac{x^4}{4} - \frac{6x^3}{3} + \frac{5x^2}{2}$ M1 A1

Evaluating at one of their x value: $\frac{1}{4} - 2 + \frac{5}{2}\left(=\frac{3}{4}\right)$ M1 A1 ft

Evaluating at the other x value: $\frac{625}{4} - 250 + \frac{125}{2}\left(=-31\frac{1}{4}\right)$ A1

$[\ldots..]_5 - [\ldots..]_1$ or $[\ldots..]_1 - [\ldots..]_5$ M1

$$-31\frac{1}{4} - \frac{3}{4} = -32$$

Total Area $= 32 + \frac{3}{4} = 32\frac{3}{4}$ A1 7

If integrating the wrong expression in (d), (e.g. $x^2 - 6x + 5$), do not allow the first M mark, but then follow scheme.

[14]

19. (a) $x^2 - 2X + 3 = 9 - x$ M1

$x^2 - x - 6 = 0$ $(x + 2)(x - 3) = 0$ $x = -2,\ 3$ M1 A1

$y = 11,\ 6$ M1 A1 ft 5

(b) $\int \left(x^2 - 2x + 3\right)dx = \frac{x^3}{3} - x^2 + 3x$ M1 A1

$\left[\frac{x^3}{3} - x^2 + 3x\right]_{-2}^{3} = (9 - 9 + 9) - \left(\frac{-8}{3} - 4 - 6\right)$ $\left(=21\frac{2}{3}\right)$ M1 A1

Trapezium: $\frac{1}{2}(11 + 6) \times 5$ $\left(=42\frac{1}{2}\right)$ B1 ft

Area $= 42\frac{1}{2} - 21\frac{2}{3} = 20\frac{5}{6}$ M1 A1 7

<u>Alternative:</u> $(9 - x) - (x^2 - 2x + 3) = 6 + x - x^2$ M1 A1

$\int \left(6 + x - x^2\right)dx = 6x + \frac{x^2}{2} - \frac{x^3}{3}$ M1 A1 ft

$$\left[6x + \frac{x^2}{2} - \frac{x^3}{3}\right]_{-2}^{3} = \left(18 + \frac{9}{2} - 9\right) - \left(-12 + 2 + \frac{8}{3}\right), = 20\frac{5}{6}$$ M1 A1, A1

[12]

20. (a) A: $y = 1$ B: $y = 4$ B1

(b) $\frac{\mathrm{d}\,y}{\mathrm{d}\,x} = \frac{2x}{25}$ $= \frac{2}{5}$ where $x = 5$ M1 A1

Tangent: $y - 1 = \frac{2}{5}(x - 5)$ $(5y = 2x - 5)$ M1 A1

(c) $x = 5y^{\frac{1}{2}}$ B1 B1

(d) Integrate: $\frac{5y^{3/2}}{3/2}$ $\left(= \frac{10y^{3/2}}{3}\right)$ M1 A1 ft

$[\,]^4 - [\,]_1 = \left(\frac{10 \times 4^{3/2}}{3}\right) - \left(\frac{10 \times 1^{3/2}}{3}\right), = \frac{70}{3}$ $(23\frac{1}{3}, 23.3)$ M1 A1, A1

<u>Alternative for (d):</u> Integrate: $\frac{x^3}{75}$ M1 A1

Area $= (10 \times 4) - (5 \times 1) - \left(\frac{1000}{75} - \frac{125}{75}\right), = \frac{70}{3}$ $(23\frac{1}{3}, 23.3)$ M1 A1, A1

In both (d) schemes, final M is scored using <u>candidate's</u> "4" and "1".

[12]

21. (a) $y = 9 - 8 - \frac{2}{\sqrt{4}} = 0 \therefore b = 4$ **(*)** B1 c.s.o. 1

(b) $\frac{\mathrm{d}y}{\mathrm{d}x} = -2 + x^{-\frac{3}{2}}$ M1

When $x = 1$ gradient $= -2 + 1 = -1$ A1

So equation of the tangent is $y - 5 = -1\,(x - 1)$ M1

i.e. $y + x = 6$ **(*)** A1 c.s.o. 4

(c) Let $y = 0$ and $x = 6$ so D is $(6, 0)$ B1 1

(d) Area of triangle $= \frac{1}{2} \times 5 \times 5 = 12.5$ B1

$$\int_1^4 (9 - 2x - 2x^{-\frac{1}{2}})\,\mathrm{d}x = \left[9x - x^2 - 2\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right]_1^4$$

Ignore limits M1 A1

$= (36 - 16 - 4 \times 2) - (9 - 1 - 4)$

Use of limits M1

$= 12 - 4$

$= 8$ A1

So shaded area is $12.5 - 8 = 4.5$ A1 6

[12]

1. To establish the x-coordinate of the maximum turning point in part (a), it was necessary to differentiate and to use $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$. Most candidates realised the need to differentiate, but the use of the zero was not always clearly shown.

Methods for finding the area in part (b) were often fully correct, although numerical slips were common. Weaker candidates often managed to integrate and to use the limits 0 and 2, but were then uncertain what else (if anything) to do. There were some attempts using y coordinates as limits. While the most popular method was to simply subtract the area under the curve from the area of the appropriate rectangle, integrating $24 - (x^3 - 10x^2 + 28x)$ between 0 and 2 was also frequently seen. Occasional slips included confusing 24 (the y-coordinate of P) with 28, subtracting 'the wrong way round' and failing to give the final answer as an exact number.

2. This question was accessible to all students and the later part differentiated between weak and strong candidates.

(a) This part of the question was generally well done with most candidates gaining both marks.

(b) Candidates had great difficulty showing that (5,4) lies on C. It was common to see numerical work, then 4 = 4 or 0 = 0 followed by no conclusion. The expectation is to see :

$x = 5$, so $y = 5^2 - 5.5 + 4$ i.e. y = 4 So (5,4) lies on the curve.

(c) A large proportion of the candidates gained full marks in this part of the question, showing that they understand the symbolisation for integration. Many included a constant of integration and some even proceeded to find a value for it via substitution, usually using the coordinate N. (Such constants were ignored.) There were very few candidates that mistakenly differentiated.

(d) There were a number of ways to find the shaded area. The easiest method was to evaluate the integral between $x = 4$ and $x = 5$. This represents an area of a region below the curve, which together with R makes up a triangle, with base of length 4 and height 4. So the area of R could then be found by subtraction. Unfortunately the area of the triangle when calculated was more likely to be:

½ × 3 × 4 or ½ × 5 × 4 rather than the correct ½ × 4 × 4.

Some chose to find the equation of the line LN and integrate, but unfortunately the limits were regularly incorrect, most commonly given as 4 and 1. There were a fair number of completely correct solutions seen but also many cases of arithmetic errors in the evaluation of integrals. Many students felt they needed to subtract a line and a curve without really considering the nature of the shapes involved in this question. Few successfully applied the alternative approaches stated on the scheme.

3. Most candidates expanded the brackets correctly and most collected to three terms although a significant number then reversed the signs before integrating. A few candidates differentiated or tried to integrate without expanding first but the majority scored the M mark here. Most substituted the correct limits and subtracted correctly, although those who evaluated f(4) and f(–1) separately often made errors in subtracting. A common mistake was the substitution of 1 instead of –1. A few split the area into two parts –1 to 0 and 0 to 4. The fraction work and the inability to cope with a negative raised to a power (here and in other questions) is quite a concern. Many candidates completed correctly and this question was reasonably well done.

4. In general, this was very well done, showing that many candidates know when to differentiate and when to integrate. In part (a), most candidates successfully differentiated the equation then presented evidence that they understood that $\frac{dy}{dx} = 0$ at a turning point. A significant number went on to find the second derivative to establish the nature of the turning point (although this was not required here, given the graph).

Most candidates integrated correctly in part (b) and a majority subtracted the area of the triangle correctly. Those who found the equation of the straight line ($y = 11x$) and then proceeded to find the area of the triangle by integration (or to subtract $11x$ from the equation of the curve and then integrate) were more likely to make mistakes than those who simply used $\frac{1}{2}bh$ for the triangle.

It was disappointing to see the final mark being lost by candidates who did not give an *exact* final answer, as specified in the question.
A few candidates who appeared to have no correct overall strategy for answering the question were still able to earn marks for their integration.

5. The first two parts were a good source of marks for most candidates. In part (c), the method of finding the area under the curve and subtracting the area under the line was the more favoured approach. In the majority of these cases the area under the line was found by calculating the area of the triangle rather than integrating, but in either case there was considerable success. Integration of the curve function was usually correct and the biggest source of error was confusion with the limits. It was surprisingly common to see

$$\int_0^6 (6x - x^2)dx - \int_0^6 2x dx \text{ (or equivalent), and } \int_0^6 (6x - x^2)dx \,[\text{or } \int_0^8 (6x - x^2)dx] - 16$$

used, and it may be that parts (a) and (b) had in some way contributed to the confusion. Candidates who subtracted the line function from the curve function before integrating often earned the marks quickly, but sign errors were not uncommon. Candidates who used longer strategies were sometimes successful but there was clearly more chance of making one of the errors noted above. Some candidates calculated the area under the curve using the trapezium rule; the mark scheme enabled them to gain a maximum of two marks.

6. Many candidates successfully expanded and integrated the given expression for y although the usual errors, such as sign slips, were seen. A few candidates took the wrong approach to the question by differentiating or using the trapezium rule. Some candidates evaluated only one integral, usually using limits of 0 and 2 for this. The majority of candidates understood the need to find two areas. Many correctly found the area under the curve between 0 and 1. However, several incorrect methods were used to find the area bounded by the curve and the x-axis between $x = 1$ and $x = 2$. These included the use of the trapezium rule and using areas of rectangles and triangles. Obtaining and explaining the negative answer to the integral between 1 and 2 and convincing examiners of a final valid answer for the total area caused some difficulties.

7. In general, candidates scored well on parts (a) and (c) of this question, usually managed part (b), but struggled with part (d). Most knew the method for part (a), and were able to differentiate correctly and solve the appropriate quadratic equation. Although part (b) asked for the value of the second derivative at A, some candidates equated the derivative $6x- 16$ to zero, solved this equation and then tried to use this result to justify the maximum. The vast majority of candidates were successful in part (c), performing the indefinite integration. Many marks were lost, however, in part (e), where candidates often had little idea how to calculate the required area. A common approach was to use limits 0 to $\frac{10}{3}$ (rather than 0 to 2), and those who continued often seemed confused as to which area they should subtract. Some supported their arguments with reference to the diagram, but more often than not triangles (or trapezia) being used were not clearly identified. For some, working was further complicated by their decision to find the area of a triangle by using the equation of a straight line, and integrating. A few produced very clear, concise and accurate methods, which were a pleasure to mark amidst the convoluted efforts of the majority.

8. The first part of this question gave difficulty to many. There are a number of possible approaches but many just wrote down $3x^{\frac{1}{2}} - x^{\frac{3}{2}} = 3\sqrt{3} - 3^{\frac{3}{2}} = 0$ and this was thought inadequate unless they could show that $3^{\frac{3}{2}}$ or $\sqrt{27}$ was $3\sqrt{3}.3^{\frac{3}{2}} = 3 \times 3^{\frac{1}{2}}$ would have been sufficient demonstration of this. In part (b), not all could solve $\frac{3}{2}x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}} = 0$ and a few found the second derivative and equated that to zero. In part (c), most could gain the first four of the five marks available but cleaning up the final answer to the single surd, $\frac{12}{5}\sqrt{3}$ or its equivalent, proved difficult and many had recourse to their calculators, which did not fulfil the condition of the question that an exact answer is to be given.

9. Almost all candidates found the x-coordinates of A and B correctly. Any errors were usually algebraic slips, but a small number of candidates compared the gradients of the two functions rather than the functions themselves.

In attempting to find the shaded area, most candidates found areas relative to the x axis, but some preferred to work relative to the y axis. Most candidates made a valid attempt at the area, although some forgot to integrate, and a few used the volume formula. The area of the trapezium was usually found separately. The answer was often correct, but some candidates said they were treating it as a triangle and others used the incorrect formula $\frac{1}{2}\left(\frac{3}{2}-\frac{1}{2}\right)(3-1)$ despite calling it a trapezium. A ln(…..) term was expected from the integration of $\frac{3}{2x}$, but many candidates did not achieve this. It was common to see either an x^2 term in the denominator or the function rewritten as $\frac{3}{2}x$ before an attempt to integrate. Candidates who recognized the correct form were sometimes confused by the constants.

A lot of correct working was seen, but several candidates gave away the final mark because they went into decimals and never gave an exact form for the final answer (as requested in the question). Candidates were expected to tidy their final answer to given a single log. term.

10. Part (a) was answered very well and most candidates scored full marks. In part (b) the majority knew they had to integrate and this was carried out accurately. The limits were usually used correctly too although arithmetic slips crept in here and those who used their calculators did not always obtain exact answers. Some forgot to subtract the rectangle and others attempted to find "line – curve" rather than "curve – line" but about 40% managed to obtain the correct answer of $\frac{1}{3}$.

11. This was often a very good source of marks for candidates, with many candidates scoring full marks. In part (a) the majority of candidates realised that differentiation was required and were able to complete the solution, although it was clear that some candidates were helped by the answer being given. However, solutions such as $x^{\frac{1}{2}} = 2 \Rightarrow x = \frac{2}{\frac{1}{2}} = 4$ were seen.

The mark scheme was quite generous in part (b) for finding the area under the curve, but in general the integration was performed well. This helped candidates score well here, even if they did not have a complete method to find the required area, or made mistakes in finding the area of the trapezium.

Candidates who found the equation of the line *AB* (some did as a matter of course but then did not use it) in order to find the area under the line, or to find the required area using a single integral, clearly made the question harder, more time consuming and open to more errors, in this case. Errors in finding the equation of the line were quite common; usually these occurred in finding the gradient or in manipulating the algebra, but it was not uncommon to see a gradient of -3 used from $\frac{dy}{dx}$ in part (a) with $x = 1$ substituted.

12. There were many very good solutions to part (a), scoring full marks or perhaps losing just one or two marks for slips in accuracy. It was disappointing, however, that so many candidates used an unnecessarily long method, finding the equation of the straight line through *P* and *Q*, then integrating, when it was much simpler to use the formula for the area of a trapezium (or equivalent). Integration techniques were usually sound, although $8x^{-2}$ was occasionally integrated to give $\frac{8x^{-3}}{-3}$. Sometimes candidates simply found the area under the curve and did not proceed any further.

Part (b) required candidates to use calculus to show that *y* was increasing for $x > 2$, and responses here often suggested a lack of understanding of the concept of an increasing function. Weaker candidates often made no attempt at this part, or simply calculated values of *y* for specific values of *x*. Usually, however, $\frac{dy}{dx}$ was found and there was some indication that a positive gradient implied an increasing function, but again a common approach was to find values of the gradient for specific values of *x*. Accurate and confident use of inequalities was rarely seen. Although there were some good solutions based on proving that $x = 2$ was a minimum, completely convincing and conclusive arguments in part (b) were rare.

13. (a) This was generally answered very well. Many candidates scored full marks, with others gaining B1, M1, A0, for answers such as $(x-3)^2-9$, or -27, or +27

(b) The majority of candidates did not use their (a) to get the answer in (b) but used differentiation. This meant that most candidates had this part correct, even if they had (a) incorrect or didn't attempt it.

(c) Again this part was well answered, especially by those who had done the differentiation in (b) as they went on to get the gradient of -6, and used (0,18) to get the equation of the line. Unfortunately, some assumed the coordinates of Q at (3,0), and found the gradient using the points A and Q, so didn't gain many marks at all in this part.

(d) Generally if candidates had answered part (c) correctly, they were able to do part (d) as well, although quite a few lost credibility because they stated that the gradient was 0. Many compared the x coordinates and deduced the line was parallel to the y axis and gained the credit.

(e) This was very well answered. Many candidates had full marks in this part even if they hadn't scored full marks earlier. A few candidates made the mistake of using the y value of 9 instead of the x value of 3 in the integral. Other errors included using a trapezium instead of a triangle, and some candidates made small slips such as the 18 being copied down as an 8 or integrating the 6x to get 6x/2, or $6x^2$. It is possible that these candidates were short of time.

14. This question was caused few problems for the majority of candidates. Part (a) was answered very well mistakes were usually simply errors with signs. A small minority ignored the instruction to "use algebra" and used a graphical calculator or some form of trial and improvement to find the coordinates of *A* and *B*, they scored no marks in part (a) but were allowed to use these results in part (b). The most popular approach in part (b) was to find the area of the trapezium and subtract the area under the curve. The integration and use of limits was nearly always carried out correctly and many correct solutions were seen. The alternative approach of subtracting first and then integrating was often attempted the wrong way around, candidates found the curve minus the line and then had to "lose" the extra minus sign at the end. Most candidates gave the exact area but some gave up this final mark in favour of a decimal approximation.

15. In part (a) of this question, some candidates failed to appreciate the need to differentiate and therefore made little progress. Such candidates often tried, in various ways, to use an equation of a straight line with gradient –2, perhaps inappropriately passing through the point *A*. Those who did differentiate were often successful, although some ignored the –2 and equated their derivative to zero.

There were many completely correct solutions to part (b). Integration techniques were usually sound, but a few candidates had difficulty in finding the correct limits, especially where they looked for a link between parts (a) and (b).

16. Although some weaker candidates made little progress with this question, most were able to pick up easy marks, particularly in parts (a) and (d). The differentiation in part (a) was completed correctly by the vast majority. In parts (b) and (c) however, many solutions showed evidence of confusion. Although a few candidates equated the *second* derivative to zero in part (b), methods were usually correct, and most candidates were able to solve the resulting quadratic equation to give $x = \frac{5}{3}$ or $x = 3$. Some then seemed to think that the point P had x-coordinate $\frac{5}{3}$ and the rest of their solution was similarly confused. Others began by finding the y-coordinate of P (12) and assuming that PQ was parallel to the x-axis, without addressing the fact that Q was a minimum. In part (c), having to show PQ to be parallel to the x-axis proved a little confusing for some, but most managed to explain about equal y-coordinates or equal gradients. Candidates should note that in a "show that…" question, a conclusion is expected. Most candidates demonstrated correct integration techniques in part (d), and most also subtracted the area of the appropriate rectangle. Apart from slips in calculation, there were many successful attempts to find the required area, scoring good marks.

17. Although there were many very good solutions to this question, a large number of candidates failed to cope with the parts requiring applications of differentiation.

Most scored the mark in part (a), either by solving an equation or by verification, for showing that $p = 6$, although arguments were occasionally incomplete.

Differentiation was required to answer parts (b) and (c) and most candidates scored the two marks for a correct derivative, seen in either of these parts. A significant number omitted part (b). Some found the gradient at A but did not proceed to find the equation of the tangent, some found the gradient of the normal, and some gave a non-linear tangent equation, failing to evaluate the derivative at $x = 6$. There was rather more success in part (c), although a common mistake here was to equate the second derivative to zero in the attempt to find the maximum turning point.

In part (d), most candidates appreciated the need for definite integration, and many completely correct solutions were seen. A few, however, used $x = 4$ (presumably taken from part (c)) instead of $x = 6$ as the upper integral limit.

18. High marks were often scored in this question. The factorisation in part (a) proved surprisingly difficult for some candidates, especially those who failed to use x as a factor. Some used the factor theorem to show that $(x – 1)$ was a factor, and then used long division, but failed to factorise $(x^2 - 5x)$. Even these, however, usually realised that 1 and 5 were the required x-coordinates in part (b). While most candidates found the gradient correctly in part (c), it was notable that others failed to realise that differentiation was needed.

Apart from arithmetic slips, the majority of candidates were able to integrate and substitute limits correctly in part (d), where the only real problem was in dealing with the negative value (region below the x-axis) for the integral from 1 to 5. Here, some tried to compensate for the negative value in unusual ways and never managed to reach an appropriate answer for the combined area.

19. The vast majority of candidates coped well with part (a), solving the equations simultaneously to find the correct coordinates of A and B. A few made it more difficult for themselves initially by finding a quadratic in y rather than x. Others simply used a table of values of x and y to find the intersection points.

The most popular method in part (b) was to find the area between the curve and the x-axis $\left(21\frac{2}{3}\right)$, and then to subtract this from the area of the trapezium, and many candidates completed this accurately to score full marks. Quite often the area of the trapezium was not considered at all, the answer being left as $21\frac{2}{3}$. Those who used integration to find the area under the straight line were more likely to make numerical mistakes. The alternative approach of subtracting first $(9 - x) - (x^2 - 2x + 3)$ and then integrating $(6 + x - x^2)$ was sometimes used successfully, but was less than fully convincing when the initial subtraction was performed "the wrong way round". Occasionally *y* limits were wrongly used for the integration instead of *x* limits.

20. Although many candidates found this question difficult, those who were competent in algebra and calculus often produced excellent, concise solutions.

Part (a) caused very little difficulty, almost everyone scoring the one available mark. In part (b), however, attempts at finding the equation of the tangent were disappointing. Some candidates made no attempt to differentiate, while others differentiated correctly to get $\frac{2x}{25}$, but failed to find the gradient at $x = 5$, using instead $\frac{2}{25}$, or even $\frac{2x}{25}$, as the gradient *m* in the equation of the tangent.

Although some had no idea what to do in part (c), the majority were able to express *x* as $\sqrt{25y}$ or equivalent. Here, it was not always clear whether the square root extended to the *y* as well as the 25, but a correct expression in part (d) gained the marks retrospectively.

The expression of *x* in terms of *y* was, of course, intended as a hint for part (d), but many candidates happily ignored this and integrated *y* with respect to *x* rather than *x* with respect to *y*. Using this approach, just a few were able to find the area of the required region by a process of subtracting the appropriate areas from a rectangle of area 40, but it was very rare to see a complete method here. Some candidates wrongly used 5 and 10 as *y* limits (or 1 and 4 as *x* limits). Those integrating 5Ö*y* with respect to *y* were very often successful in reaching the correct answer.

21. No Report available for this question.